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Advanced course on

HIGH RESOLUTION ELECTRONIC MEASUREMENTS
IN NANO-BIO SCIENCE

INSTRUMENTATION FOR NOISE MEASUREMENTS

Noise as Signal

Marco Sampietro

Outline of the lesson

We have been FIGHTING all along the course AGAINST NOISE !!

Noise is <u>universal</u> and <u>unavoidable</u> (thermal noise, shot noise, 1/f noise, ...)

In electronic devices & circuits it is important to measure the noise …

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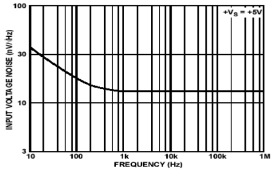
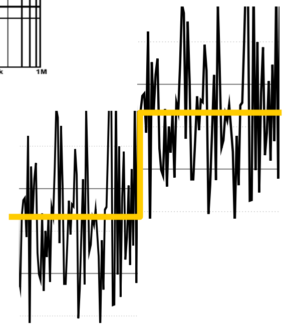


… as it sets the minimum detectable signal.

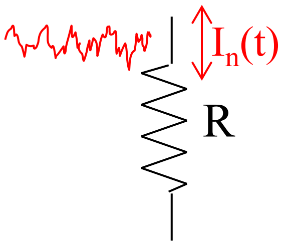


Can NOISE be OUR FRIEND ?

Use it as a macroscopic «echo» of the micro-physics ruling the device properties

How to MEASURE very low NOISE ?

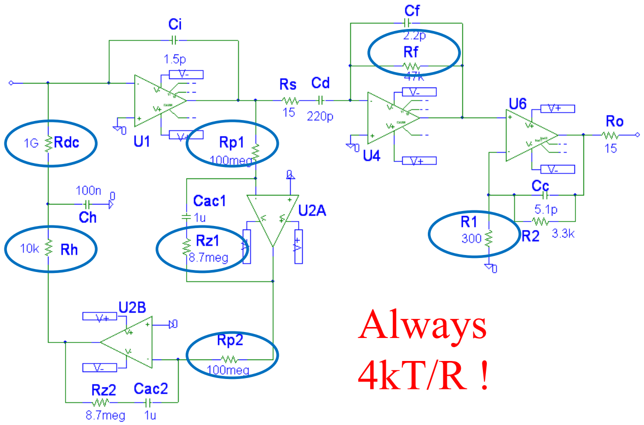
What is the noise of a resistor?



For **any resistor** (dissipative system) in thermal equilibrium :

$$S_i = \frac{4kT}{R}$$

The thermal noise is commonly used also in *non-equilibrium* conditions



Always 4kT/R !

Why ?

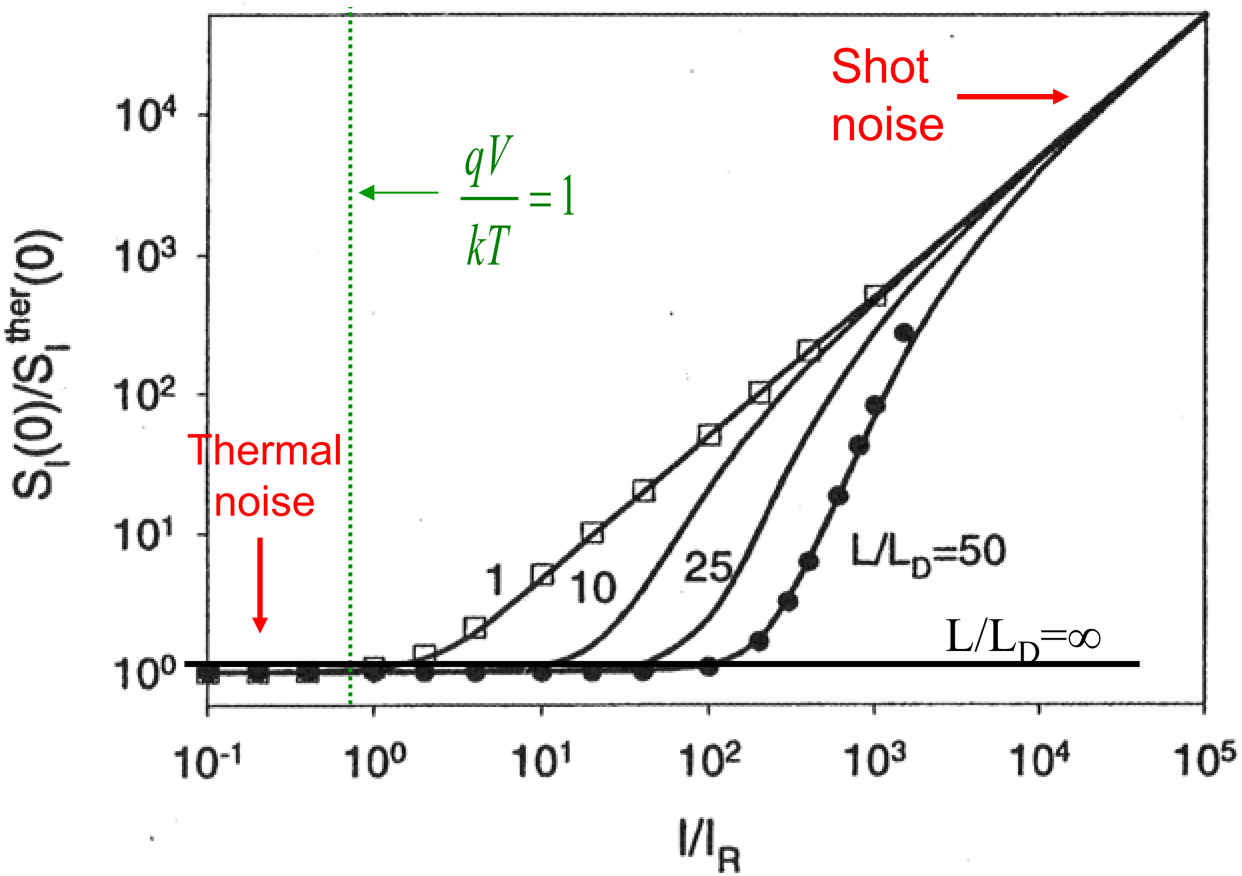
Inelastic scattering, by dissipating the energy that electrons gain from the field and randomizing the momentum, reduces the electron average energy to that of the lattice, and therefore the noise corresponds to thermal-Nyquist noise at any bias.

Why we do not see any granularity of the charge (shot noise) ?

$$S_i=2qI \quad [A^2/Hz]$$

The prediction of the G-R theory

G.Gomila, L.Reggiani, *Phys. Rev*. B62 (2000), p.8068



Departure from the thermal noise:

$$I_1 = \left(\frac{L}{L_D}\right)^{4/3} I_R \qquad I_R = \frac{V_{th}}{R}$$

($L>L_D$; per $L<L_D \rightarrow I_1=I_R$)

Shot noise:

$$I_2 = \left(\frac{L}{L_D}\right)^{2} I_R \qquad (\tau_T = \tau_d)$$

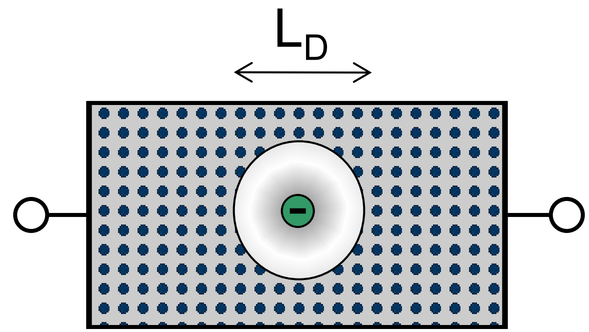
($L>L_D$; per $L<L_D \rightarrow I_2=I_R$)

A macroscopic resistor might show shot noise !

Key parameters

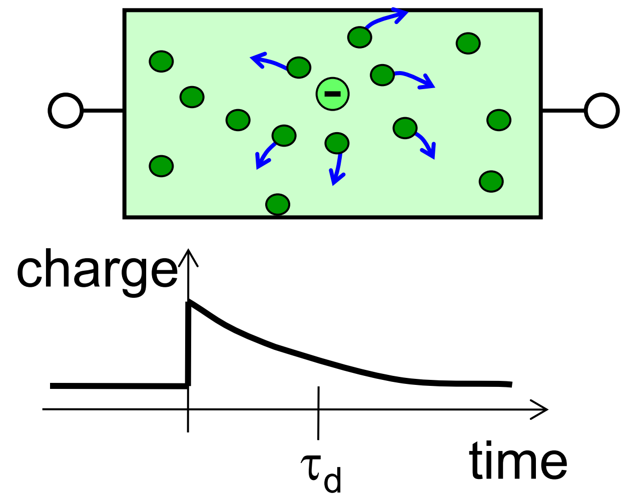
$$L_D = \sqrt{\frac{\varepsilon V_{th}}{q\ n}}$$

Debye length



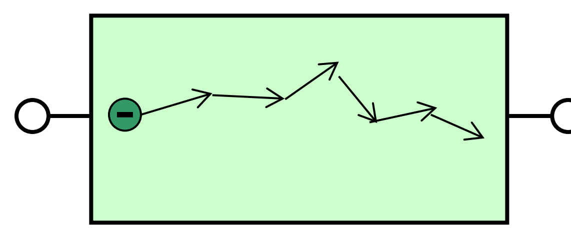
$$\tau_d = \frac{\varepsilon}{q\ n\ \mu}$$

Dielectric relaxation time



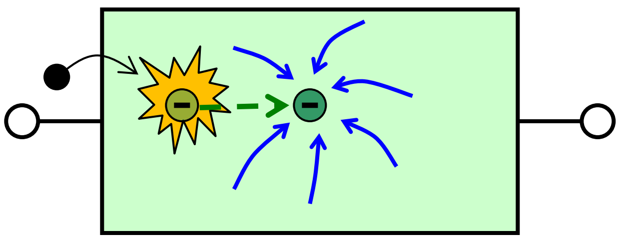
$$\tau_T = \frac{L^2}{\mu V}$$

Transit time



Thermal noise in resistors ("standard" case)

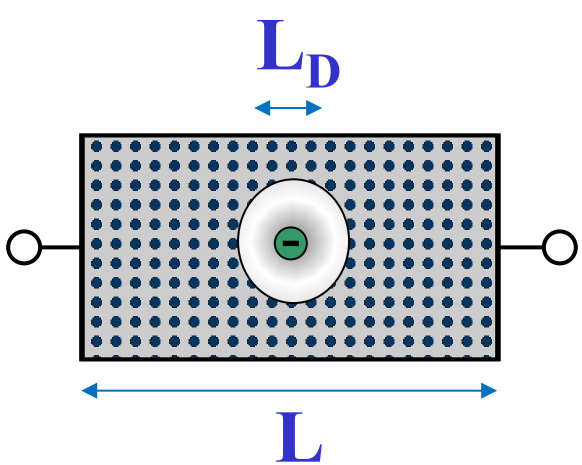
transit time ≫ τ_d (dielectric relaxation time)



Time long enough to shield the carrier
→ electrodes "do not see" the single carrier
but a "collective" effect

Thermal noise

Alternatively :

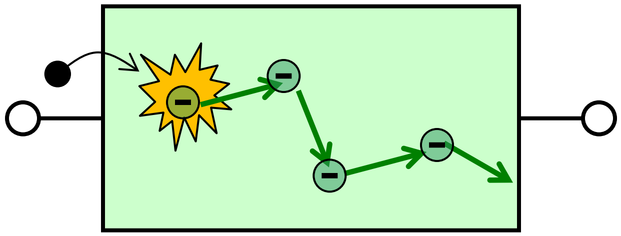


$L >> L_D$

The carrier is shielded
→ electrodes **"do not see"** the single carrier

Shot noise in resistors

transit time << τ_d (dielectric relaxation time)

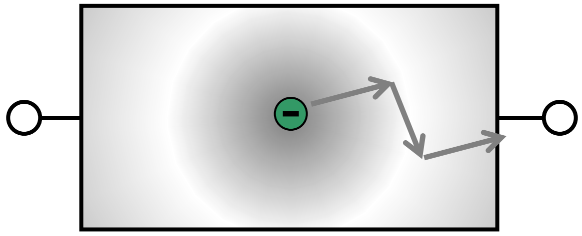


The material has no time to shield the carrier → electrodes "see" the single charge

(mean time btwn collisions) $\tau_m \ll$ transit time

Shot noise

$L_D \gg L$

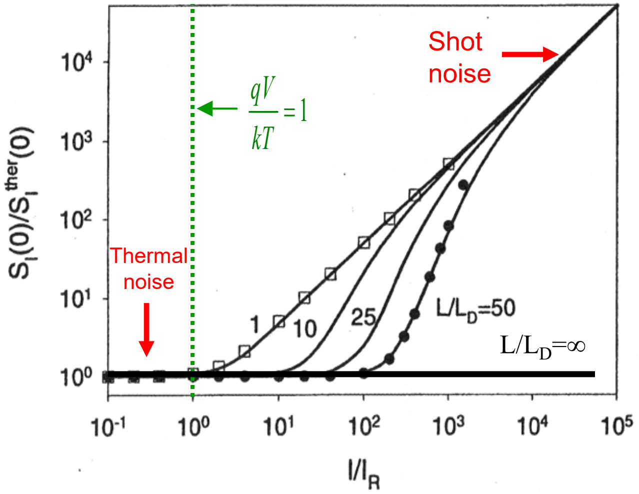


Charge carriers are independent.
→ electrodes "**see**" the single carrier.

Fluctuation of the number of carrier due to random scattering

Independent of the injection of carriers: it is the random motion in the material that gives the shot noise (**carriers thermalize but do not correlate**)

Examples

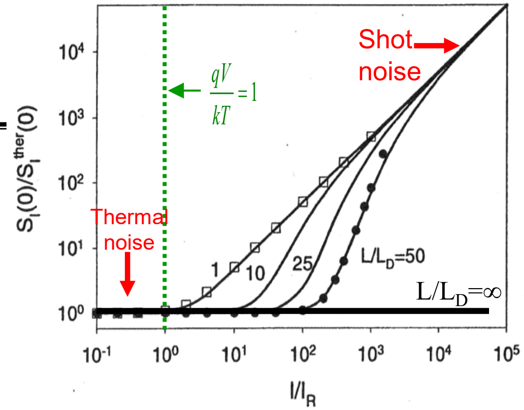


- **"Standard" resistor:**
  R=100kΩ
  L=1mm
  ➡ L_D= 0.5nm
  Thermal-shot transition at *E=10 GV/mm !!!*

- **Heavily doped silicon resistor:**
  n= 10^{17} cm^{-3}
  L=1mm
  ➡ L_D=12.5nm
  Thermal-shot transition at *E= 9 kV/mm* *(8V/μm)*

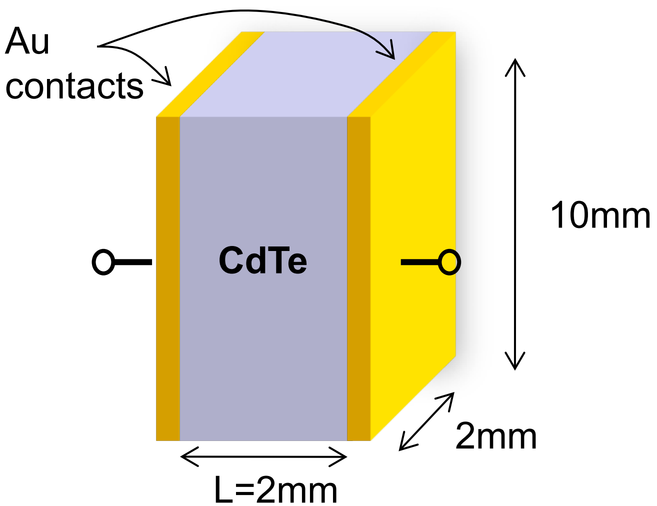
- **Lightly doped silicon resistor:**
  n= 10^{14} cm^{-3}
  L=1μm
  ➡ L_D=400nm
  Shot noise at ***E= 1.6 kV/cm (160mV/μm)***
  (transition at 850 V/cm)

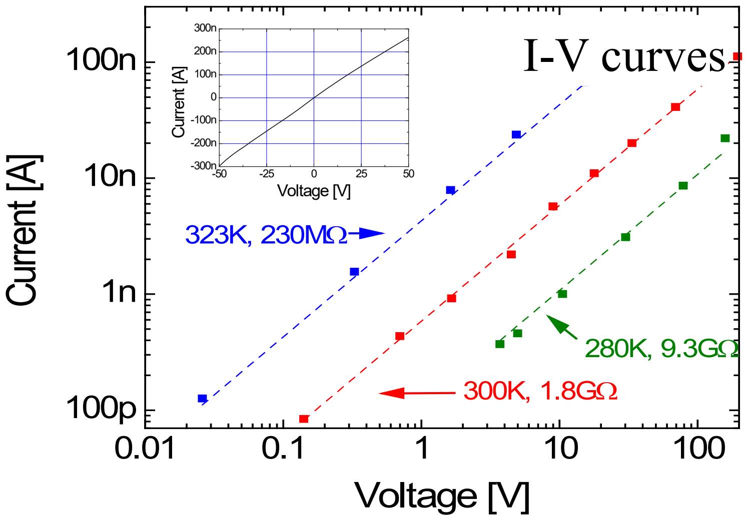
Experimental validation on CdTe



CdTe crystal (ohmic contact in gold):

- Wide band gap (1.47eV)
- Lightly doped ($p \cong 9 \cdot 10^7$ cm$^{-3}|_{T=300K}$
- □ ρ=1.8GΩ·cm (L/L_D=4.4)
- Mobility is electric field independent up to tens of kV/cm



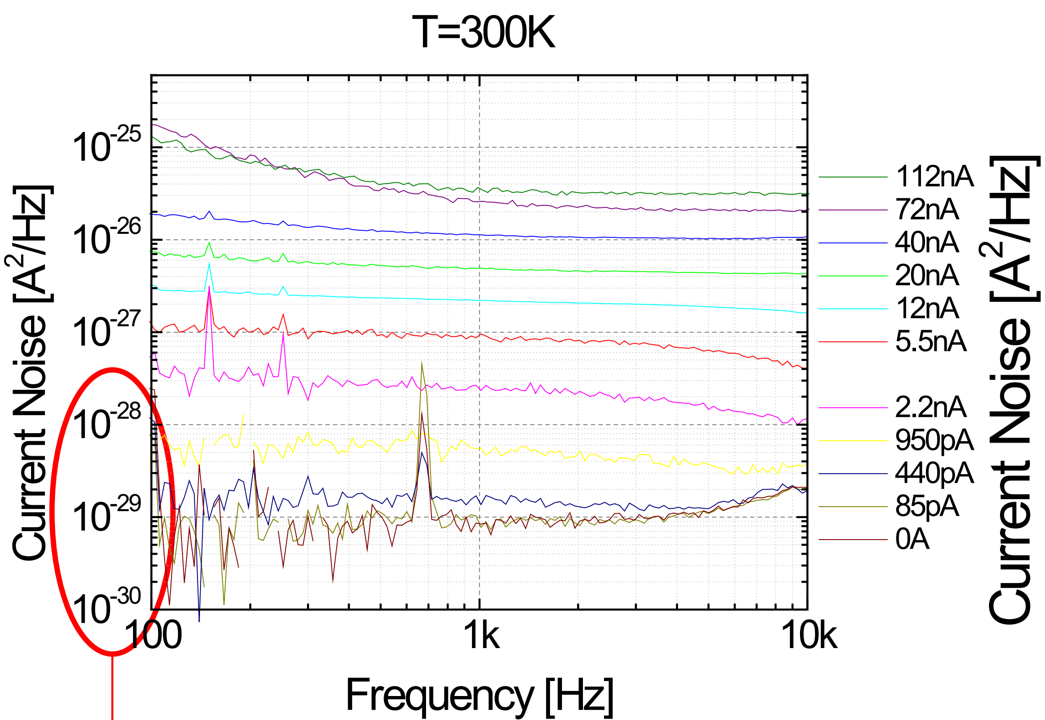


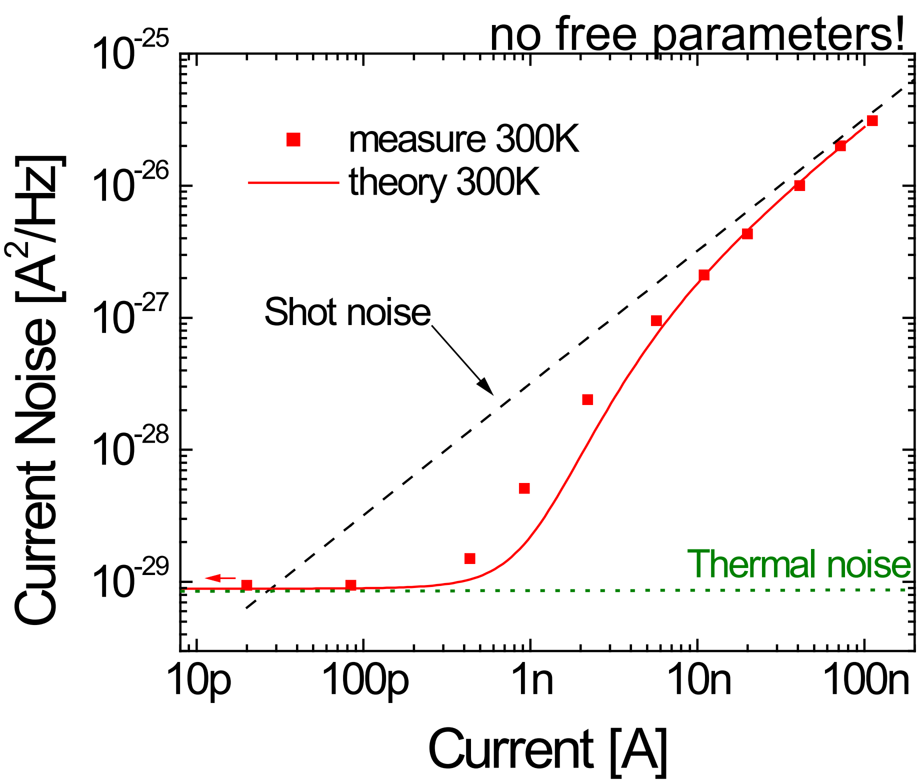
Macroscopic!

Good linearity (and symmetry) at different temperatures

Exponential dependence on the temperature ($n = n_0 \exp(-E/kT)$)

Shot noise of a resistor (at room temperature)





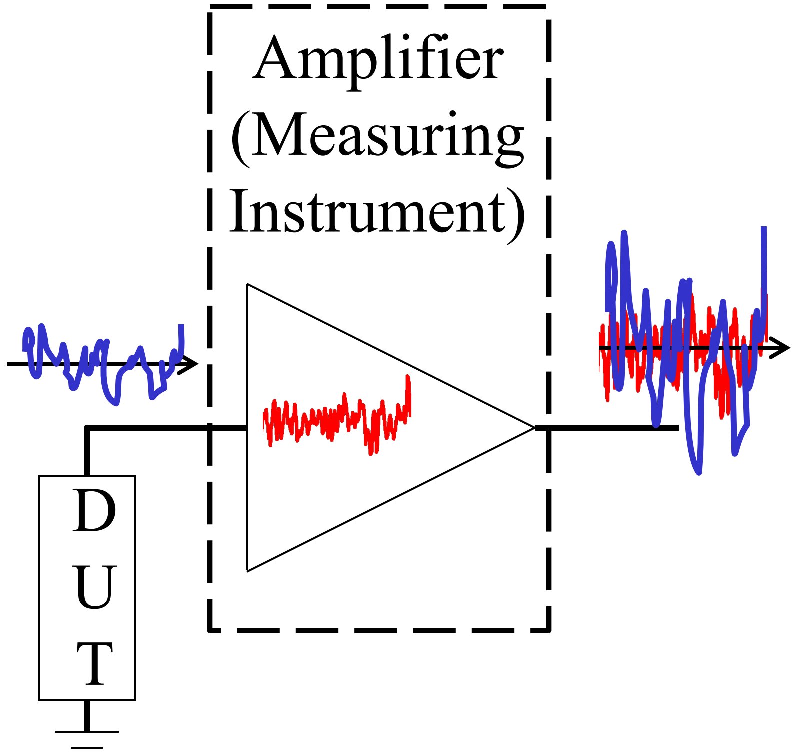
Very small values of noise !

T [K]	p [cm^{-3}]	L/L$_D$
280.5	2 x 10^7	2
300	9 x 10^7	4.4
323	7 x 10^8	11.8

G. Ferrari et al., APL, 83, 2450 (2003)

G. Gomila et al., PRL, 92, 226601 (2004)

How to measure very low noise ?

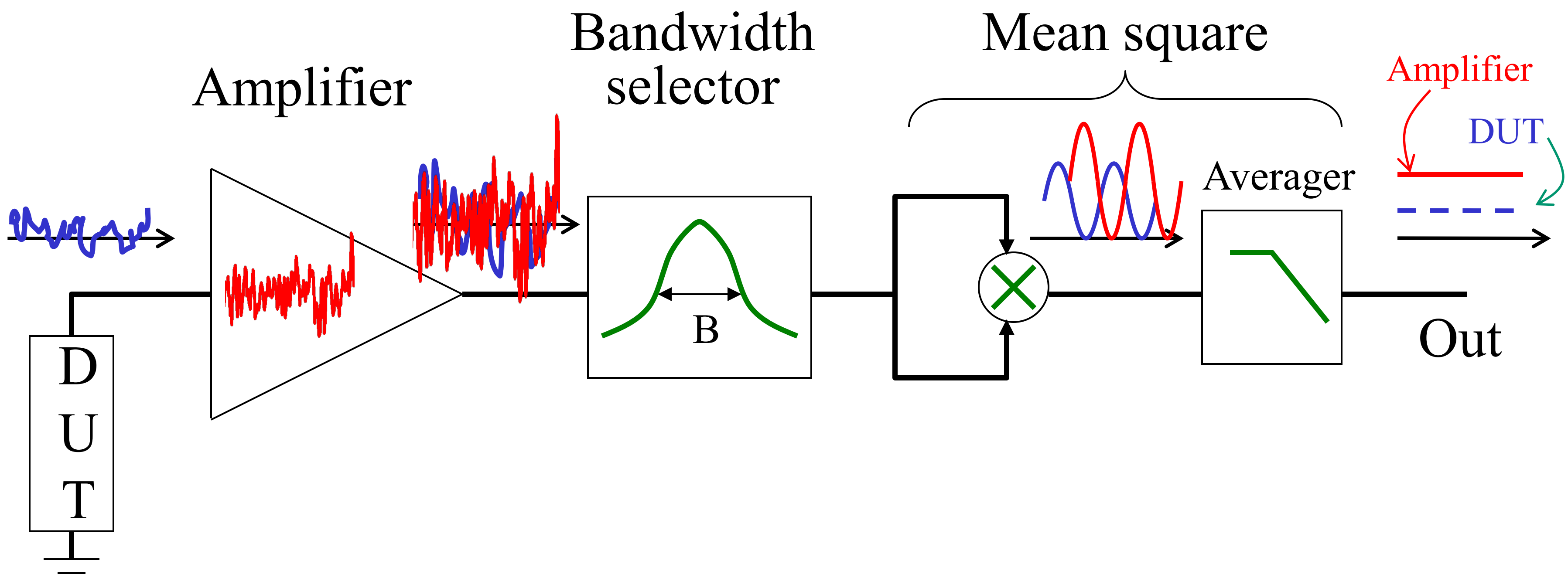


Noise power of input amplifier **adds** to the DUT signal power

and therefore sets the minimum detectable “DUT NOISE” signal.

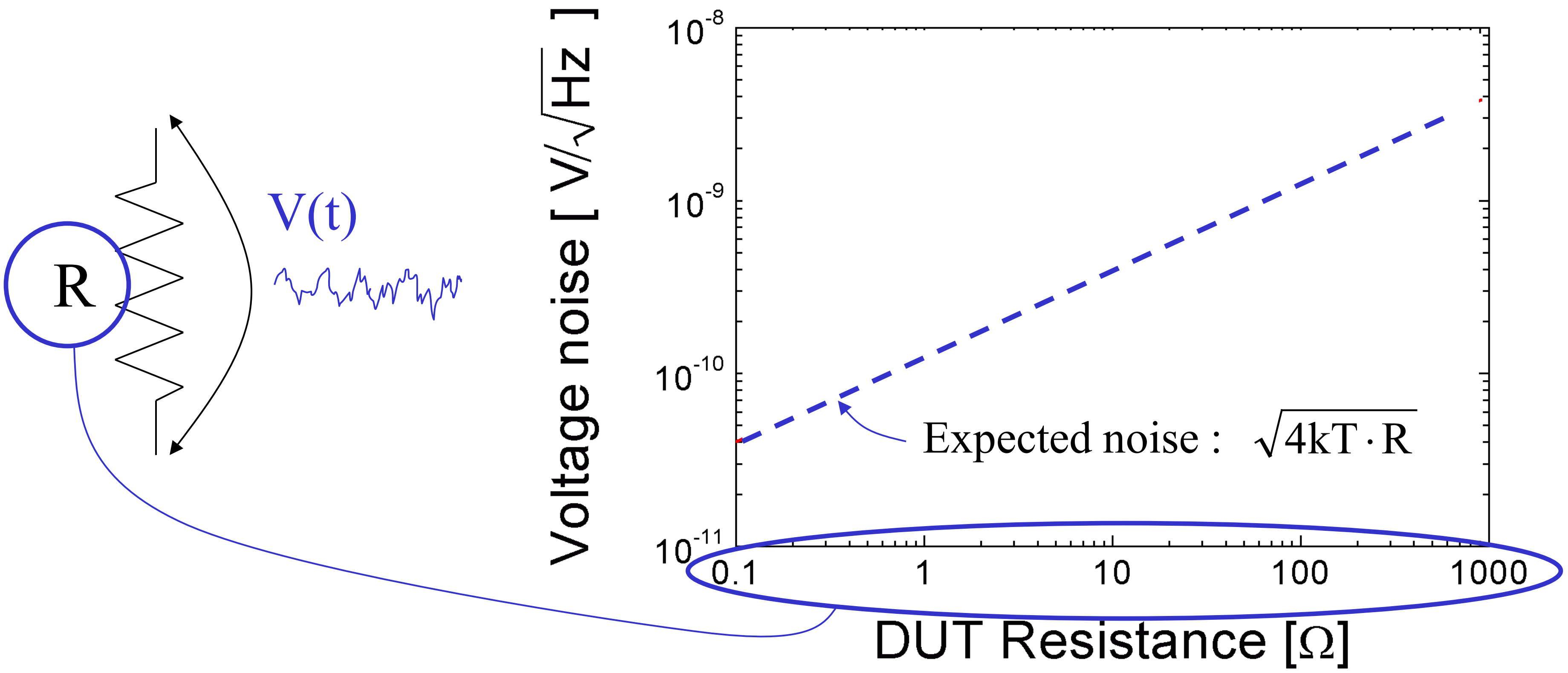
This apply frequency by frequency

A standard spectrum analyzer

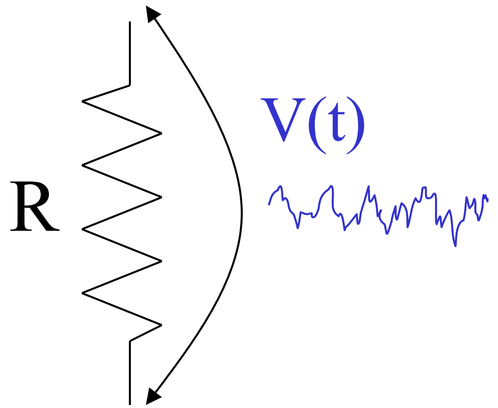


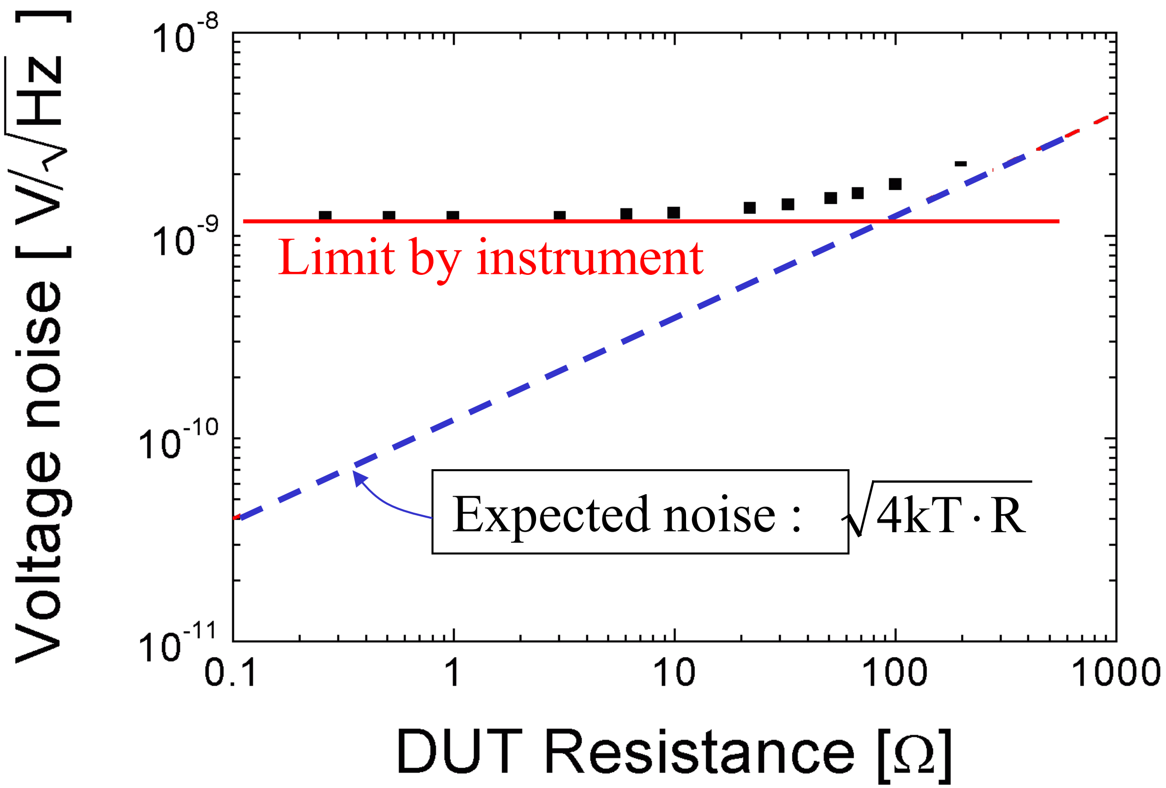
Typical white noise power of best commercial analyzers is 1nV/√Hz.
(10^{-18} V^2/Hz)

An example


R
V(t)
Voltage noise [V/$\sqrt{Hz}$]
10^{-8}
10^{-9}
10^{-10}
10^{-11}
0.1
1
10
100
1000
DUT Resistance [Ω]
Expected noise : $\sqrt{4kT \cdot R}$

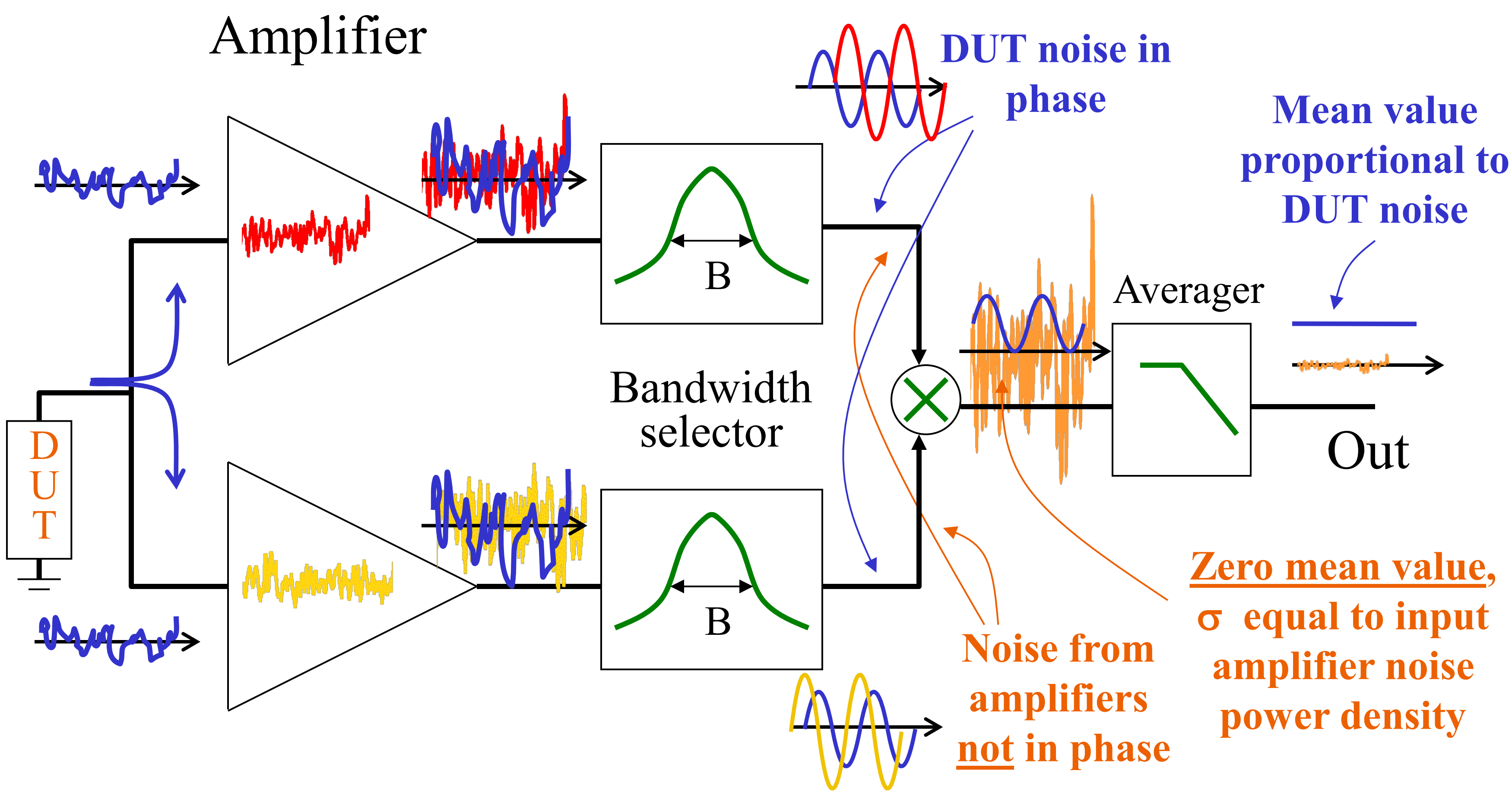
An example





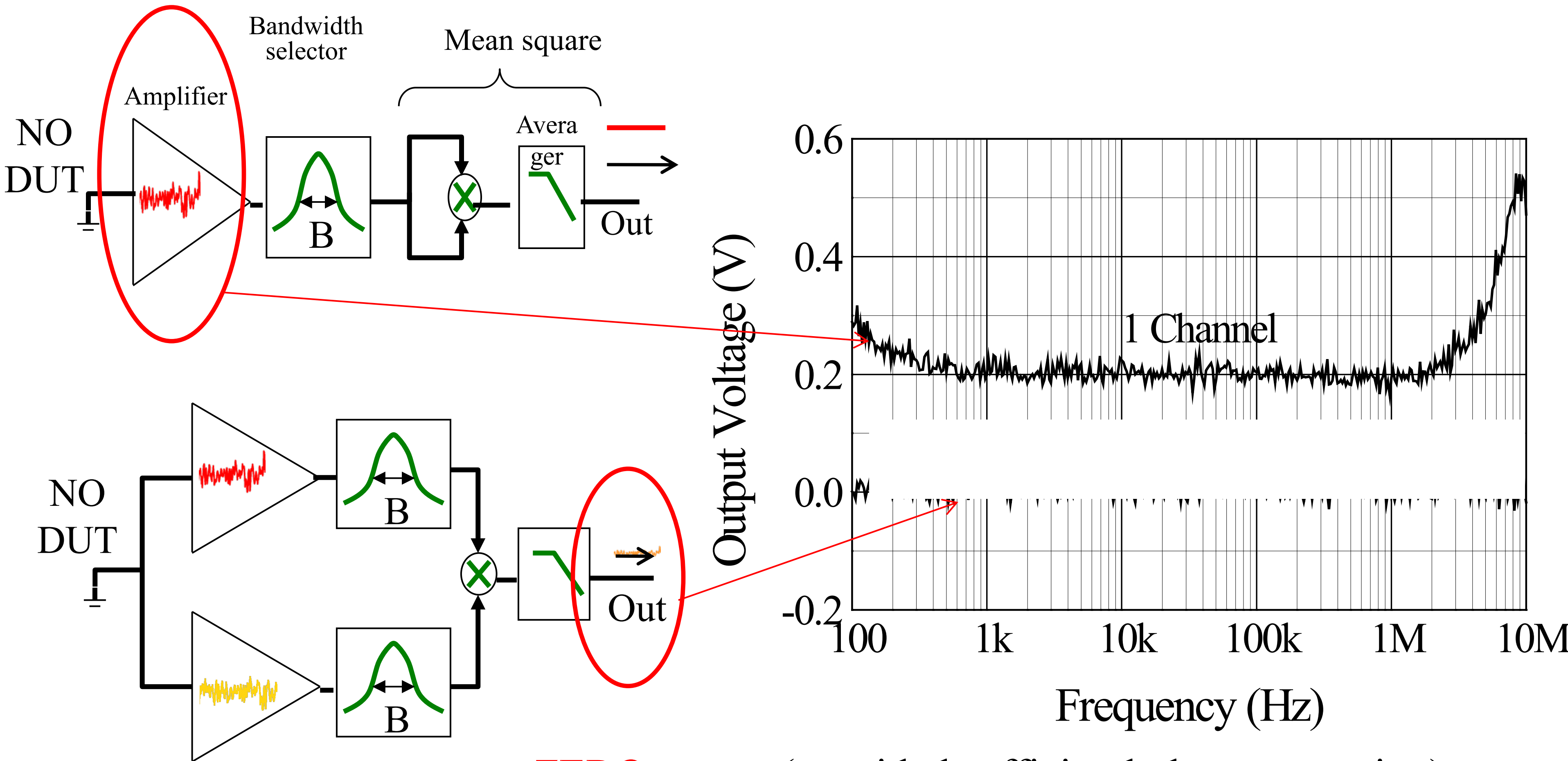
No chance to measure smaller noises ?

A two channels scheme



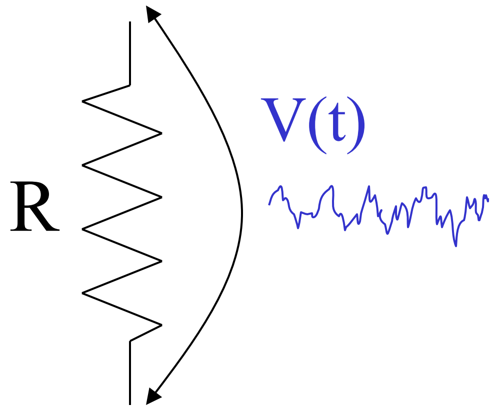
Comparison btwn the two instruments

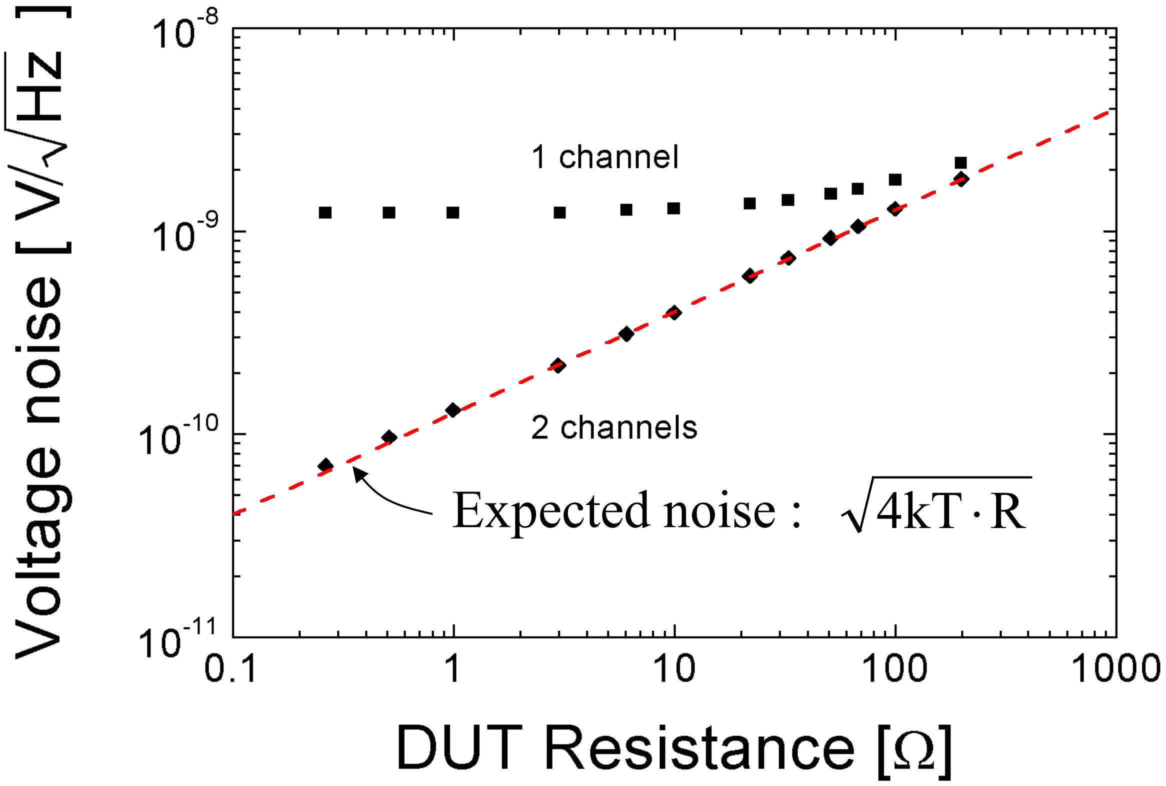
NO signal applied (NO input DUT)



ZERO output (provided sufficiently long avaraging)
irrespective to the type of input noise (1/f, white, f)

An example

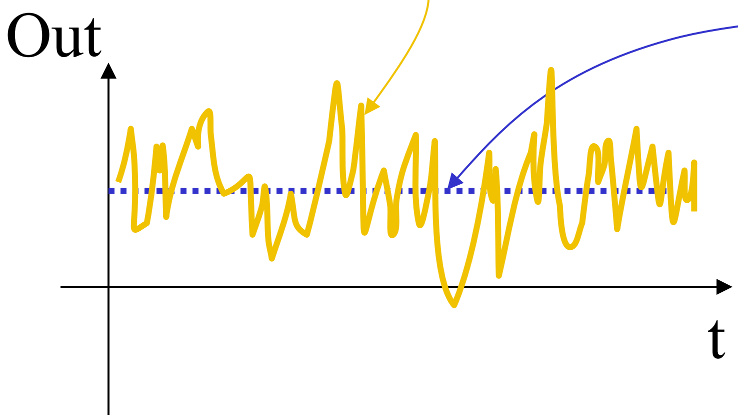




M.Sampietro et al. , *Review of Sci. Instrum.*, Vol.70, n.5, 2520-2525 (1999)

Sensitivity of the instrument

The level of fluctuation around the DUT value [V^2] is :



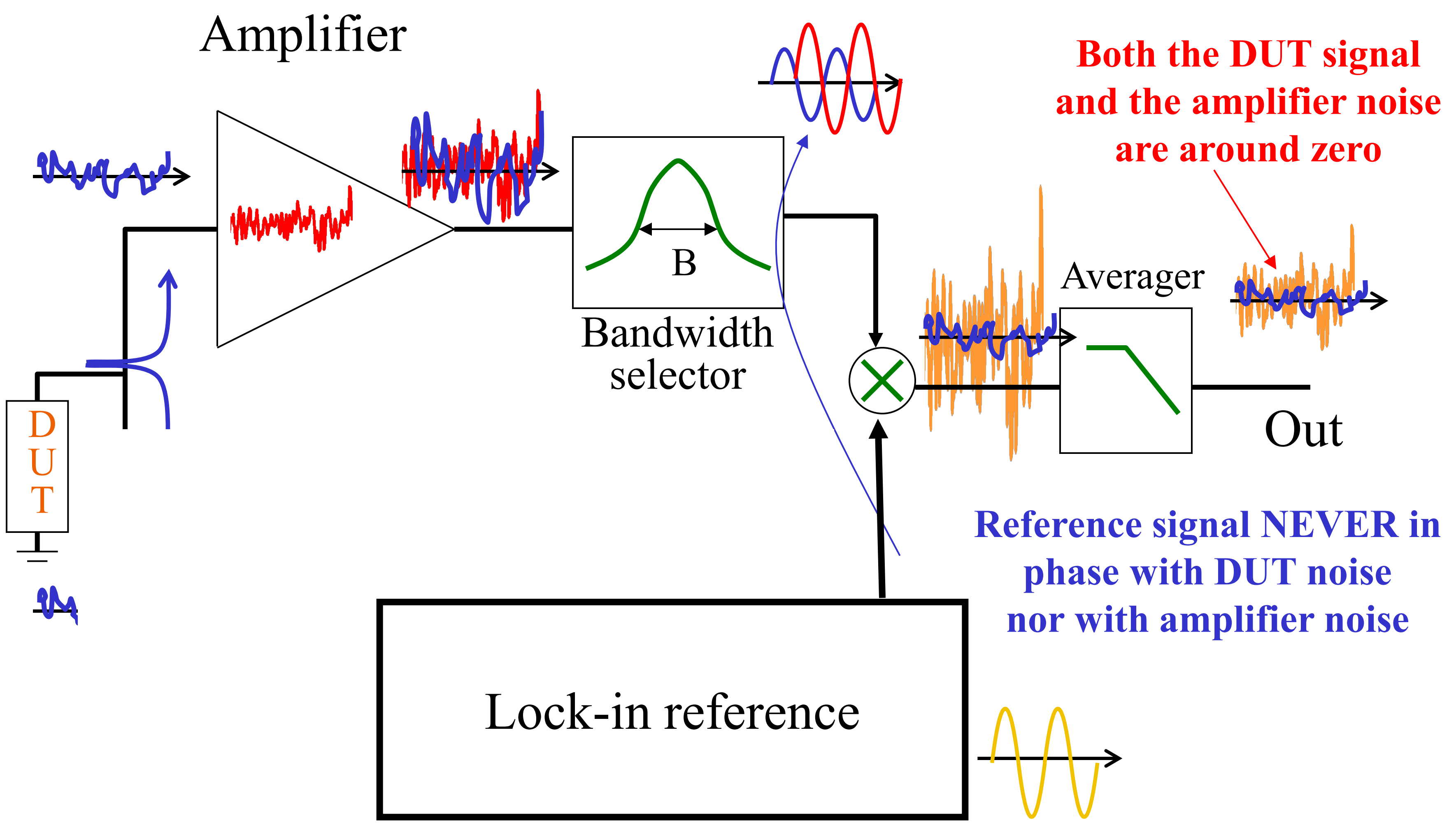
$$\sigma_{S_{DUT}} = s_n^2 \cdot \frac{1}{\sqrt{2 \cdot T_m \cdot \Delta f}}$$

Instrument *sensitivity* is imposed by:

- the amplifier input noise
- the available time for the measurement
- the desired frequency resolution

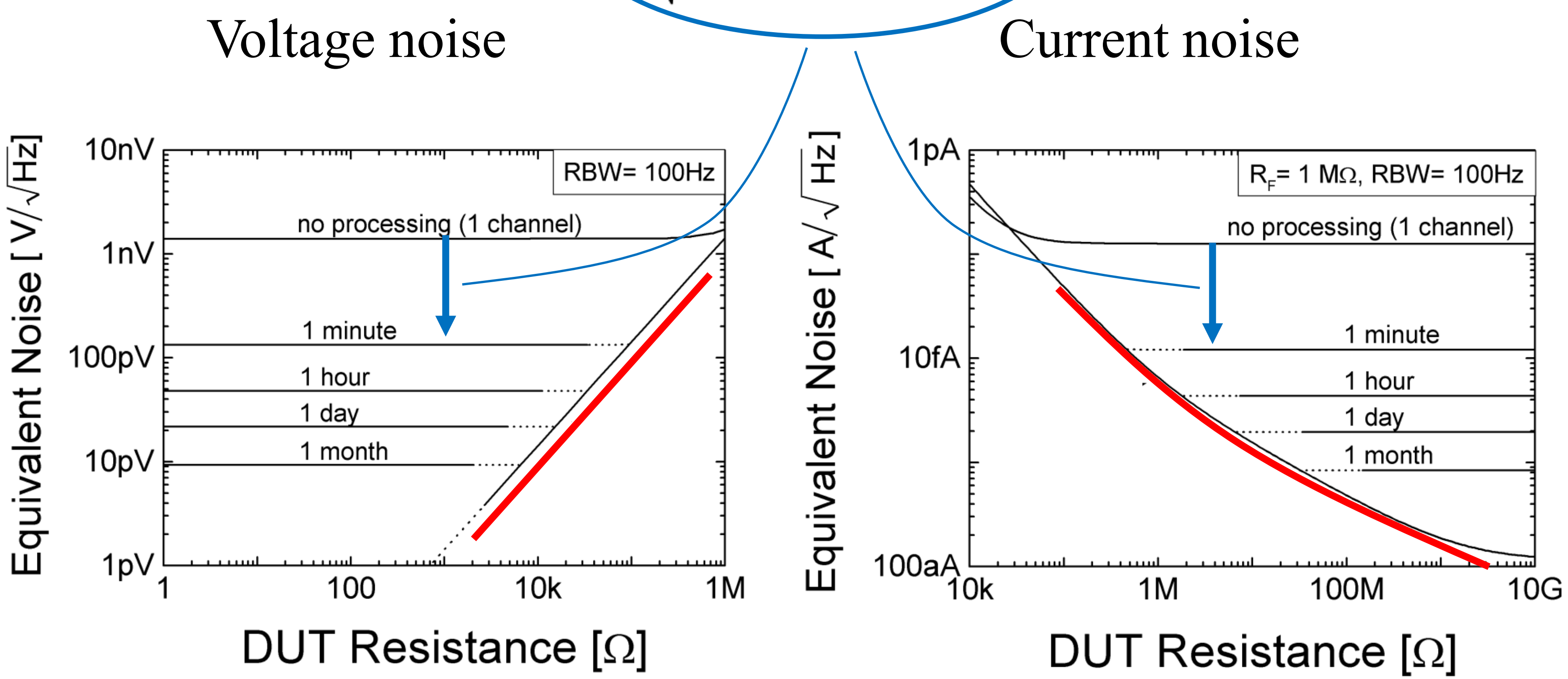
Note that σ_{SDUT} [V^2/Hz] or [A^2/Hz] gives the amount of noise power obtained with T_m and/or Δf. The noise level in [V/√Hz] or [A/√Hz] is obtained as $\sqrt{\sigma_{SDUT}}$.

NOT possible with a standard lock-in



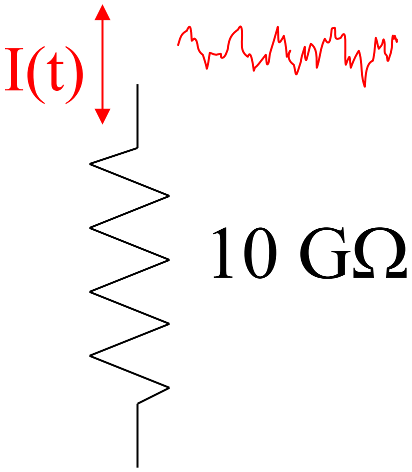
Sensitivity limit vs measuring time

$$\sqrt{S_n^2 \cdot \frac{1}{\sqrt{2 \cdot T_m \cdot \Delta f}}}$$



Limits imposed by residual correlations (see additional material)

Example of current measurement



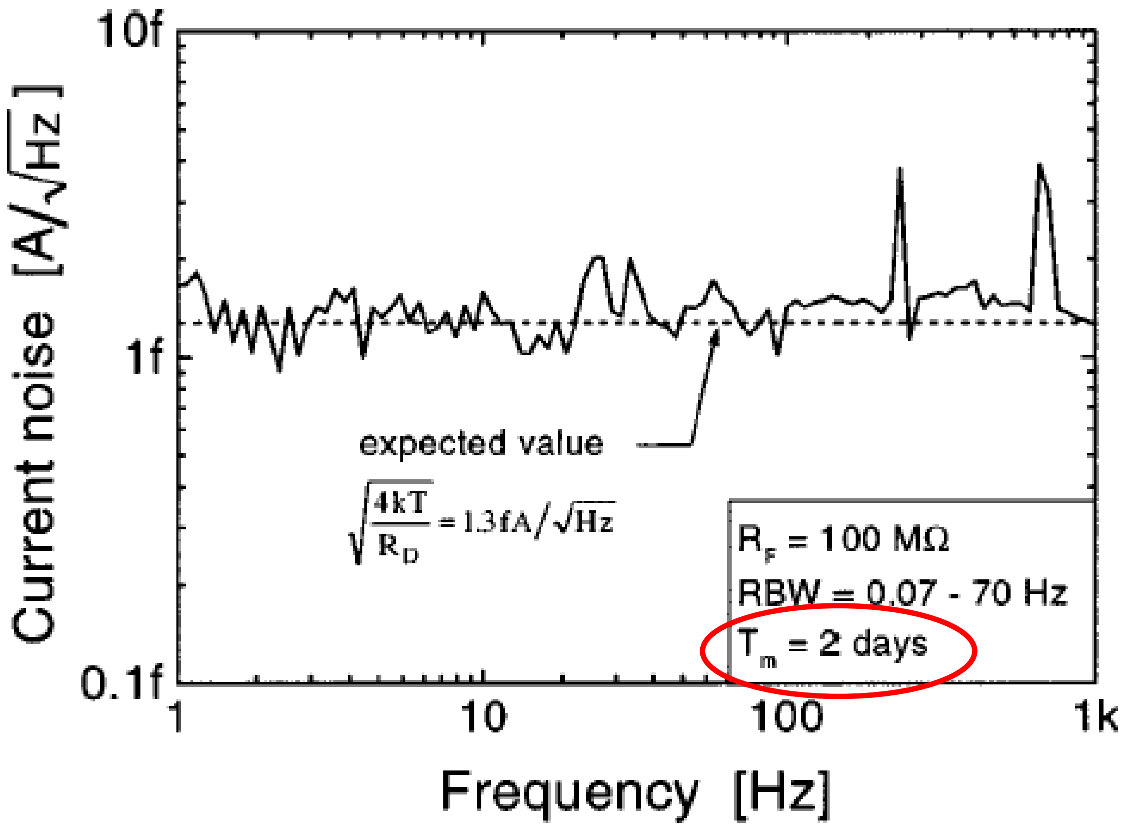
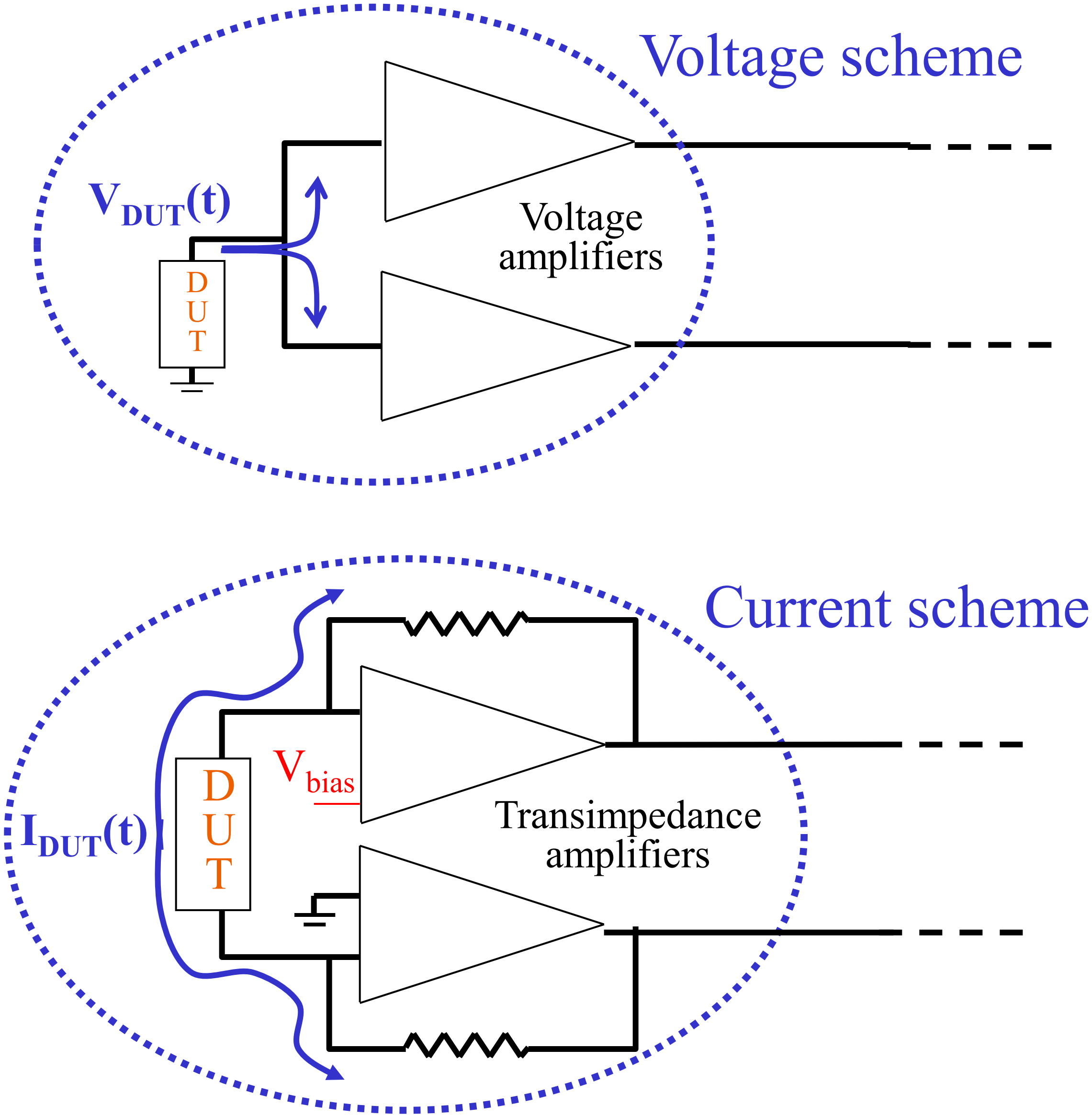


FIG. 7. Frequency spectrum of the current noise produced by a resistor of 10 GΩ. Peaks are probably due to an imperfect shielding from interferences that produce correlated signals.

M.Sampietro et al. , *Review of Sci. Instrum.*, Vol.70, n.5, 2520-2525 (1999)

2 possible front-ends



Current *vs* Voltage scheme

Choice dictated by DUT impedance to minimize residual correlations:

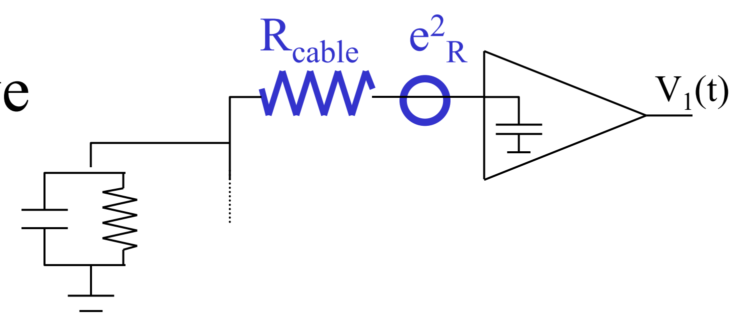
- low impedance DUT (< 10 kΩ) ➡ voltage mode
- high impedance DUT (> 10 kΩ) ➡ current mode

Current scheme has practical advantages :

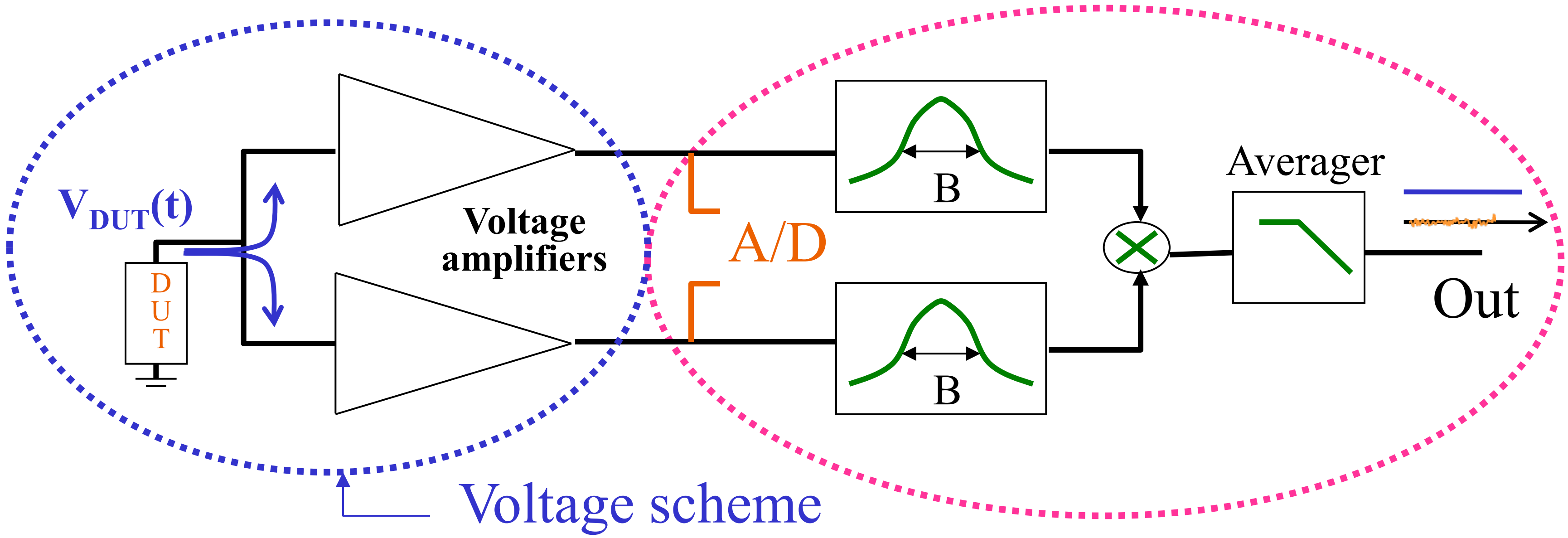
- DUT can be bias directly by the instrument
- DUT biasing network produces less correlated noise

Voltage scheme :

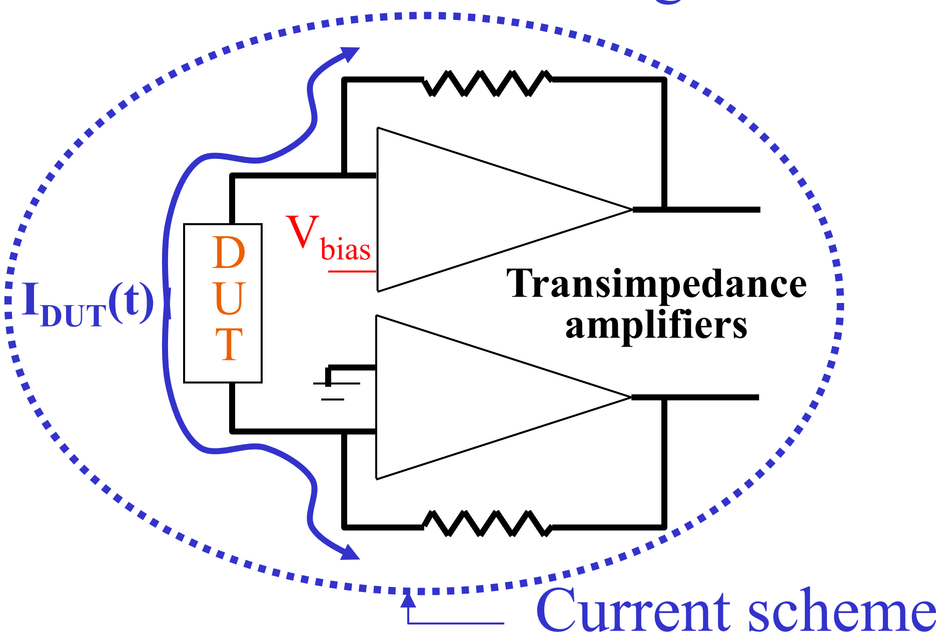
- Noise from cables are less effective



Practical realisation of the instrument



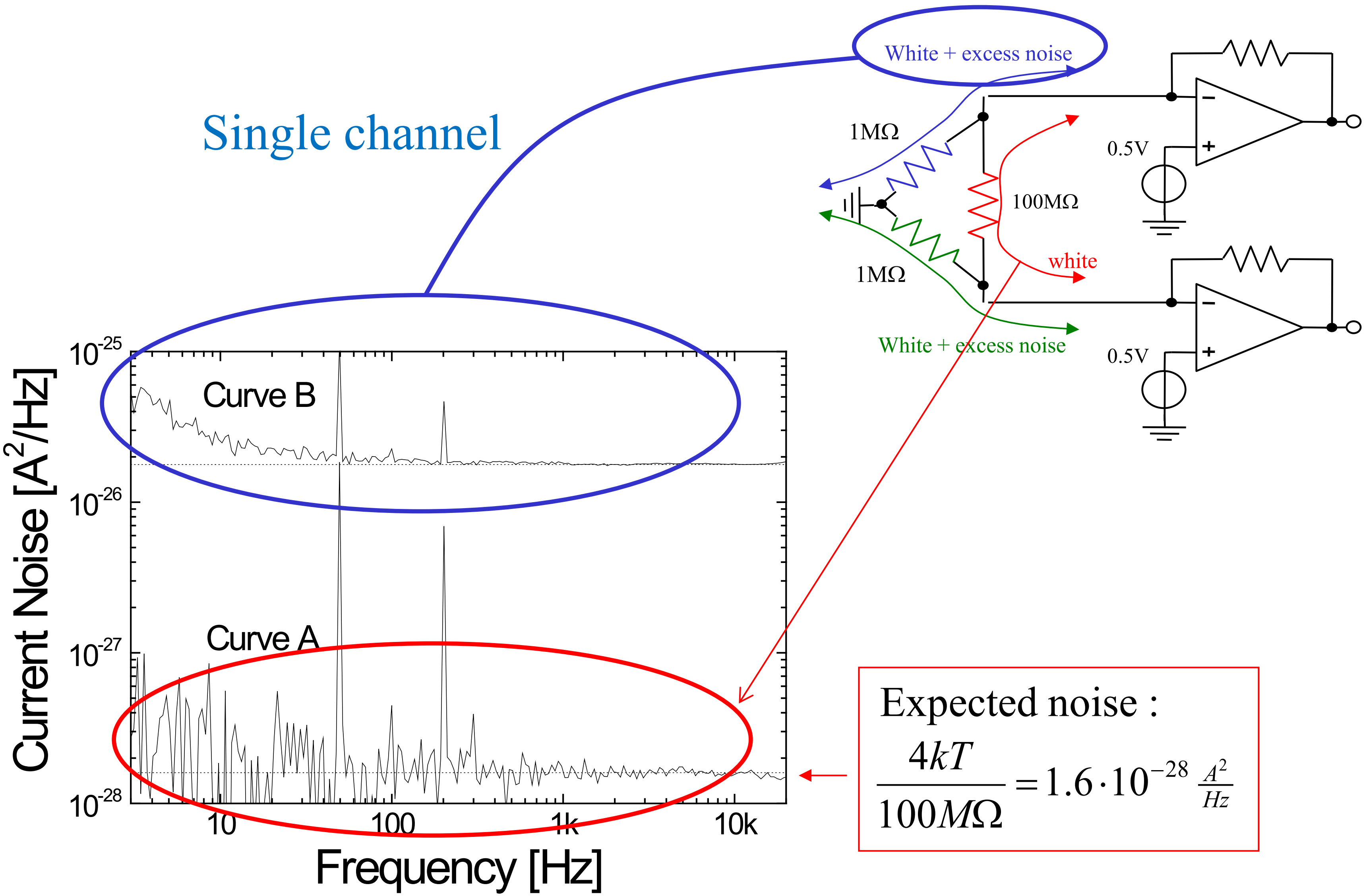
Analog processing :
1 point of the spectrum acquired at a time (needs N measurements)



Digital processing :
ALL points of the spectrum acquired at the same time (*eq.* to N filters in parallel)

Extraction of single noise from multipoles

Single channel

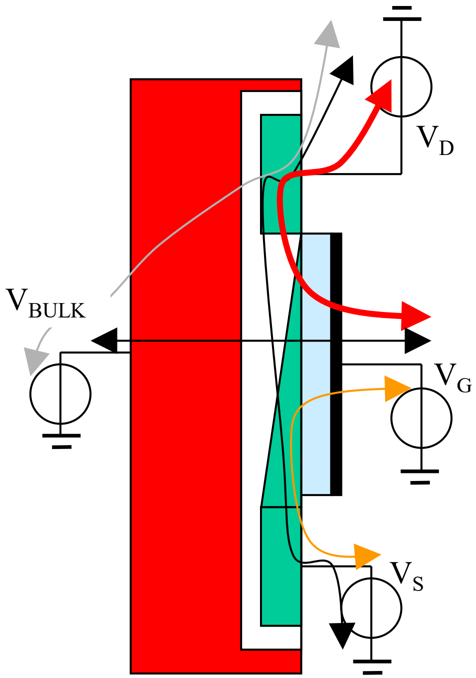


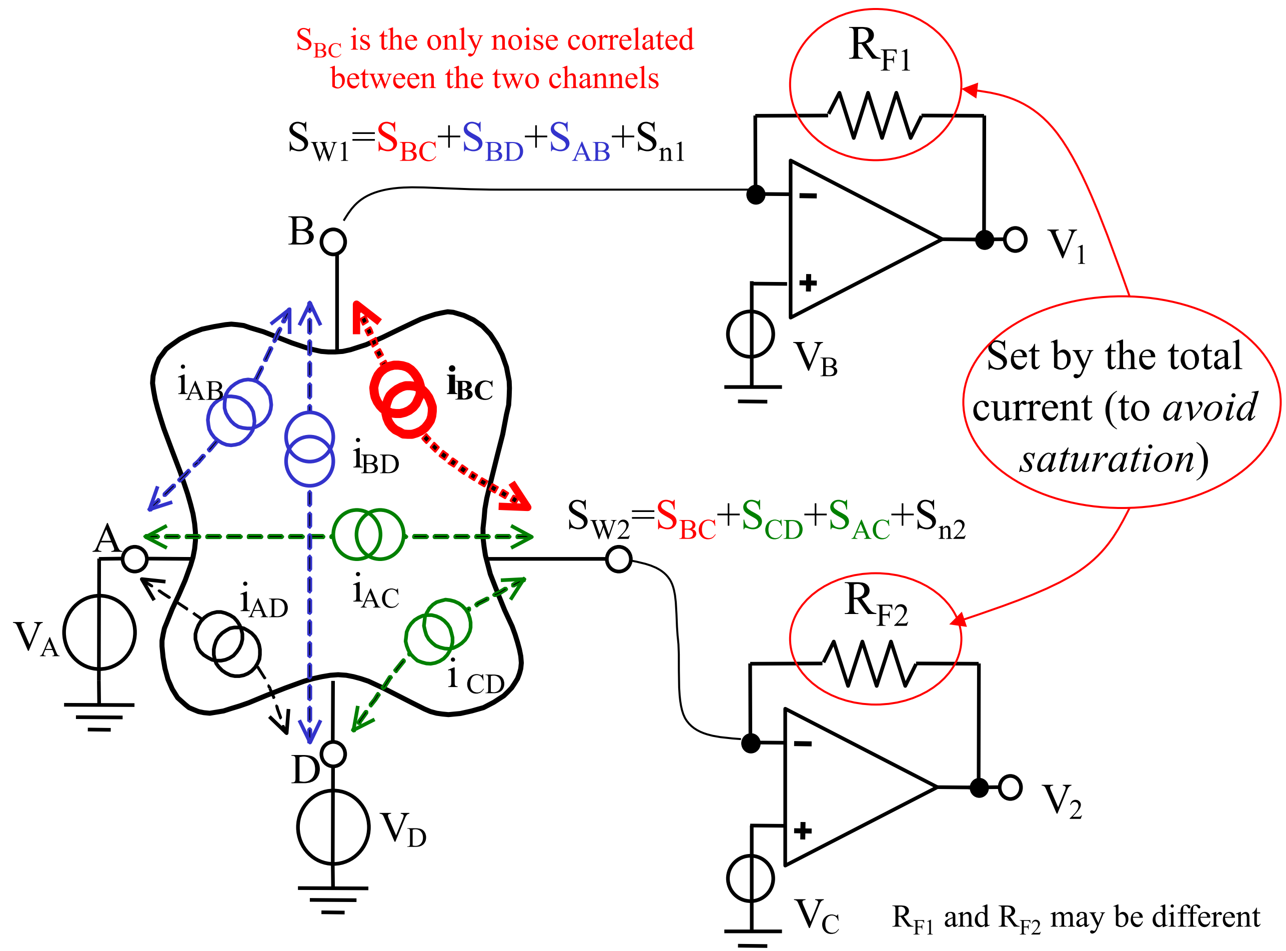
Expected noise :

$$\frac{4kT}{100M\Omega} = 1.6 \cdot 10^{-28} \ \frac{A^2}{Hz}$$

G.Ferrari et al. , *Review of Sci. Instrum.*, Vol.75, n.12, 5367-5369 (2004)

Extraction of single noise from multipoles





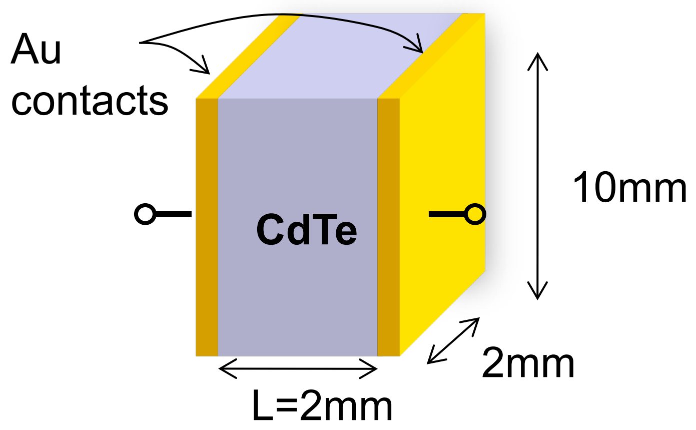
$$T_m \approx \frac{1}{2 \cdot BW} \frac{S_{W1} \cdot S_{W2}}{S_{BC}^2}$$

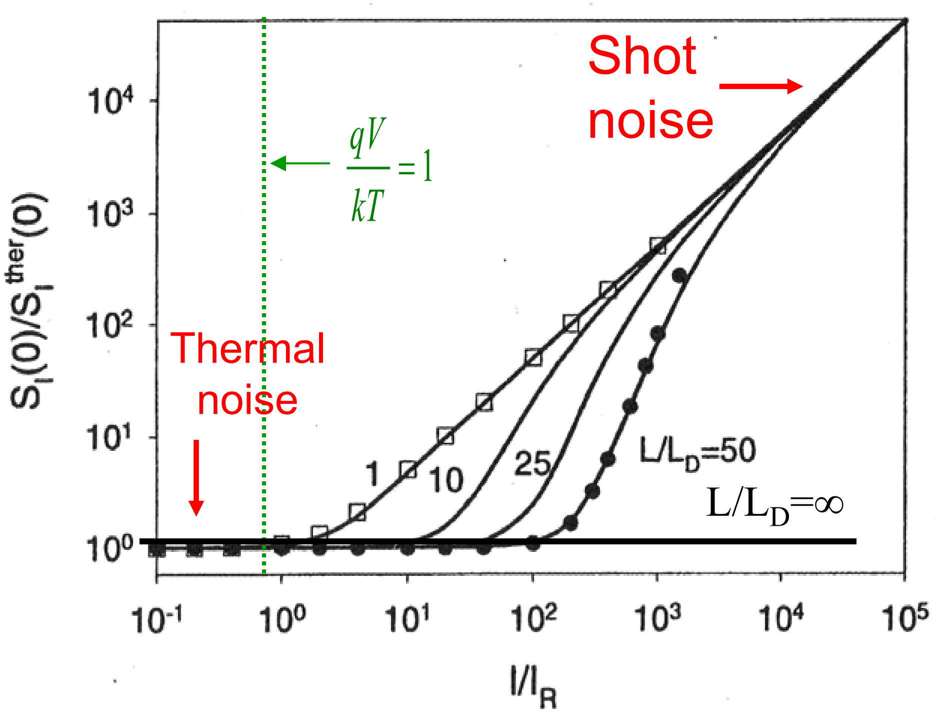
Ex : $S_{W1}=S_{W2} = 100\ S_{BC}$, BW = 100 Hz } $T_m \sim 2$ min

In summary …

Things to remember (1)

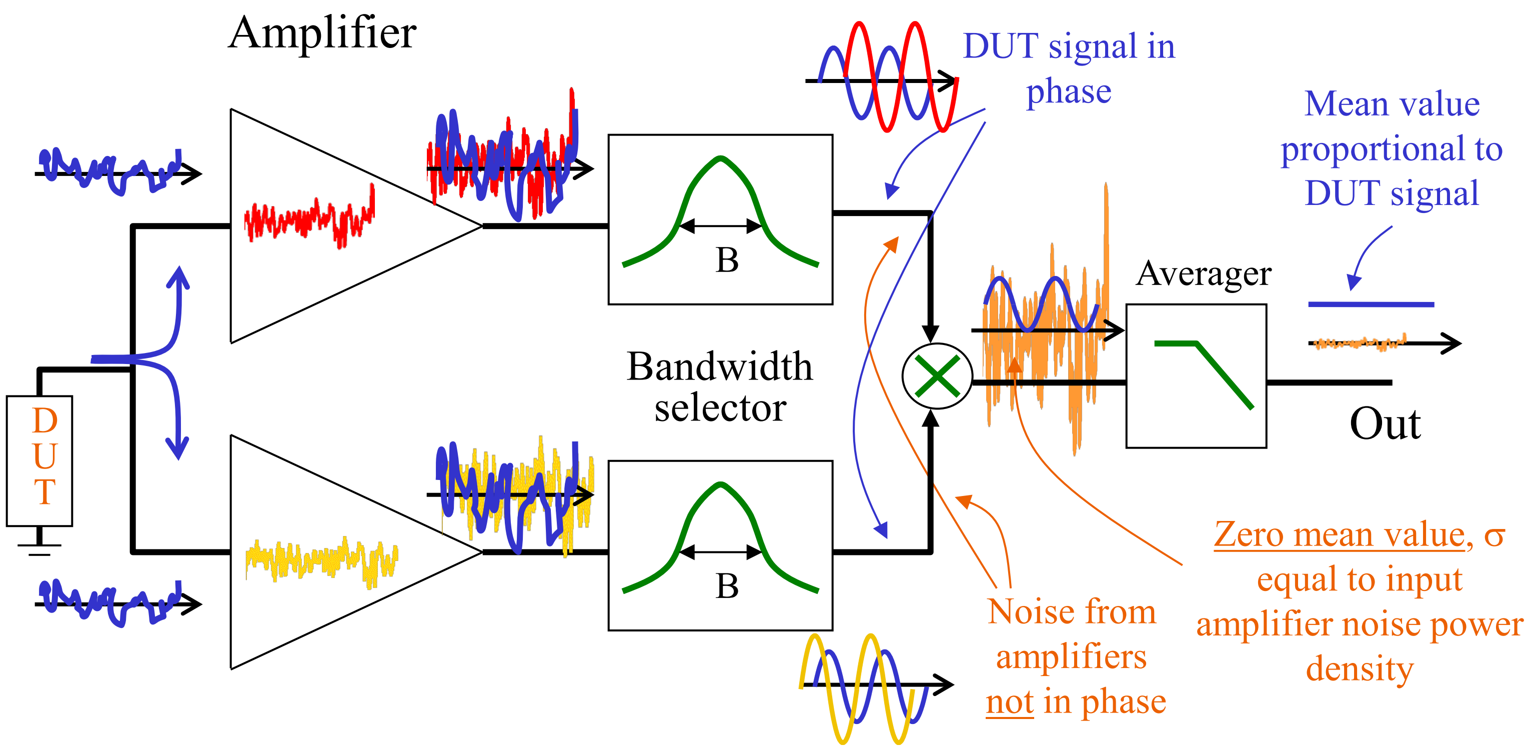
Noise can be an interesting «signal»





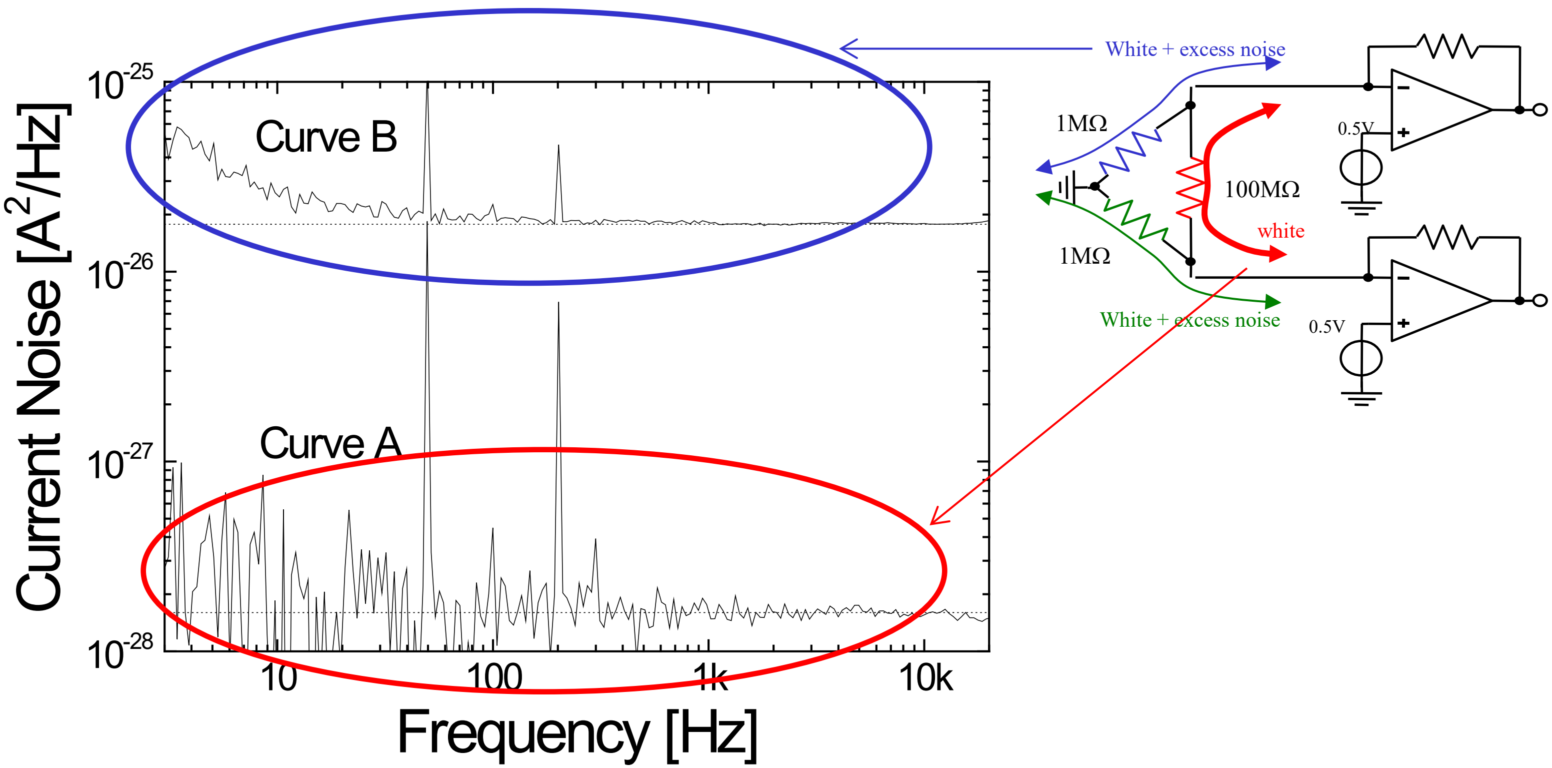
Things to remember (2)

Very small noise can be measured by using a Correlation Spectrum Technique



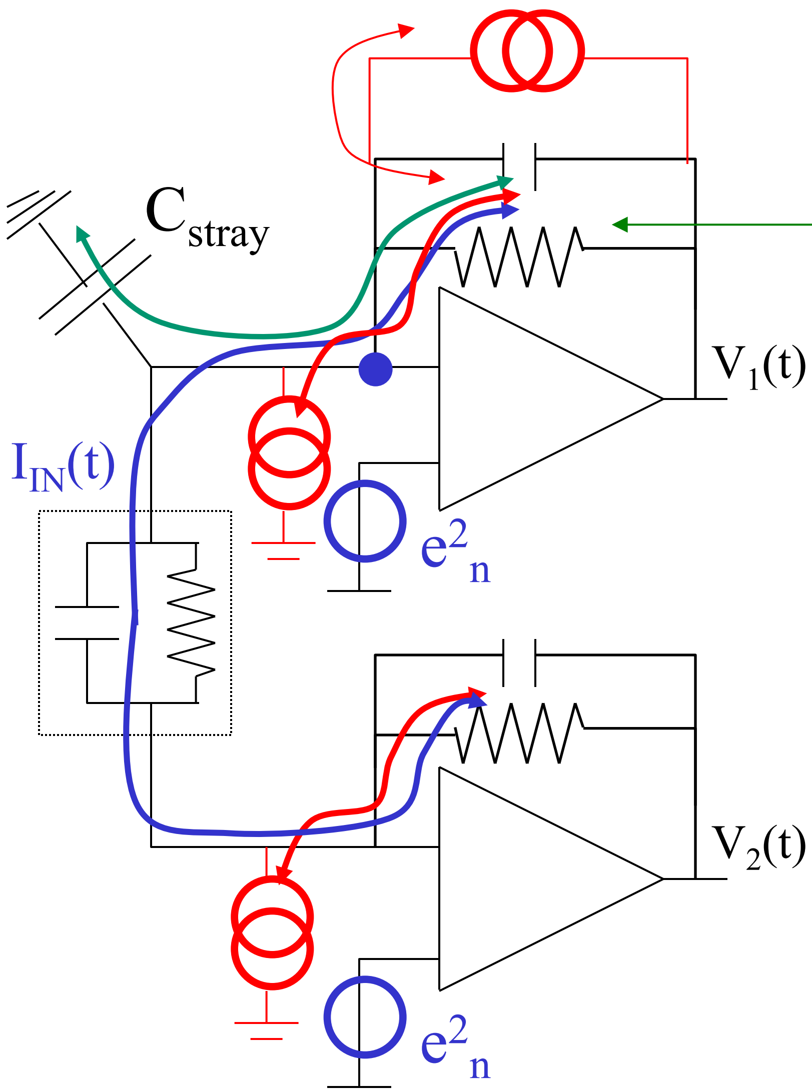
Things to remember (3)

Sorting a noise among many others is possible



ADDITIONAL MATERIAL

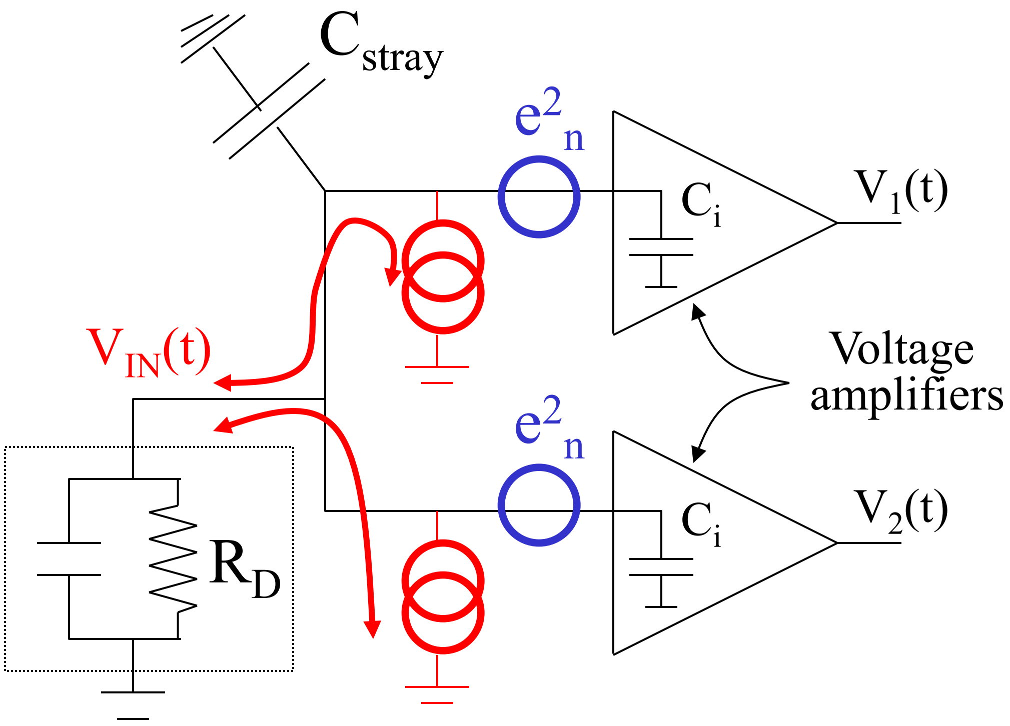
Residual correlations : current scheme



R_F is limited by:
- dinamic range of $I_{DUT}|_{DC}$
- bandwidth

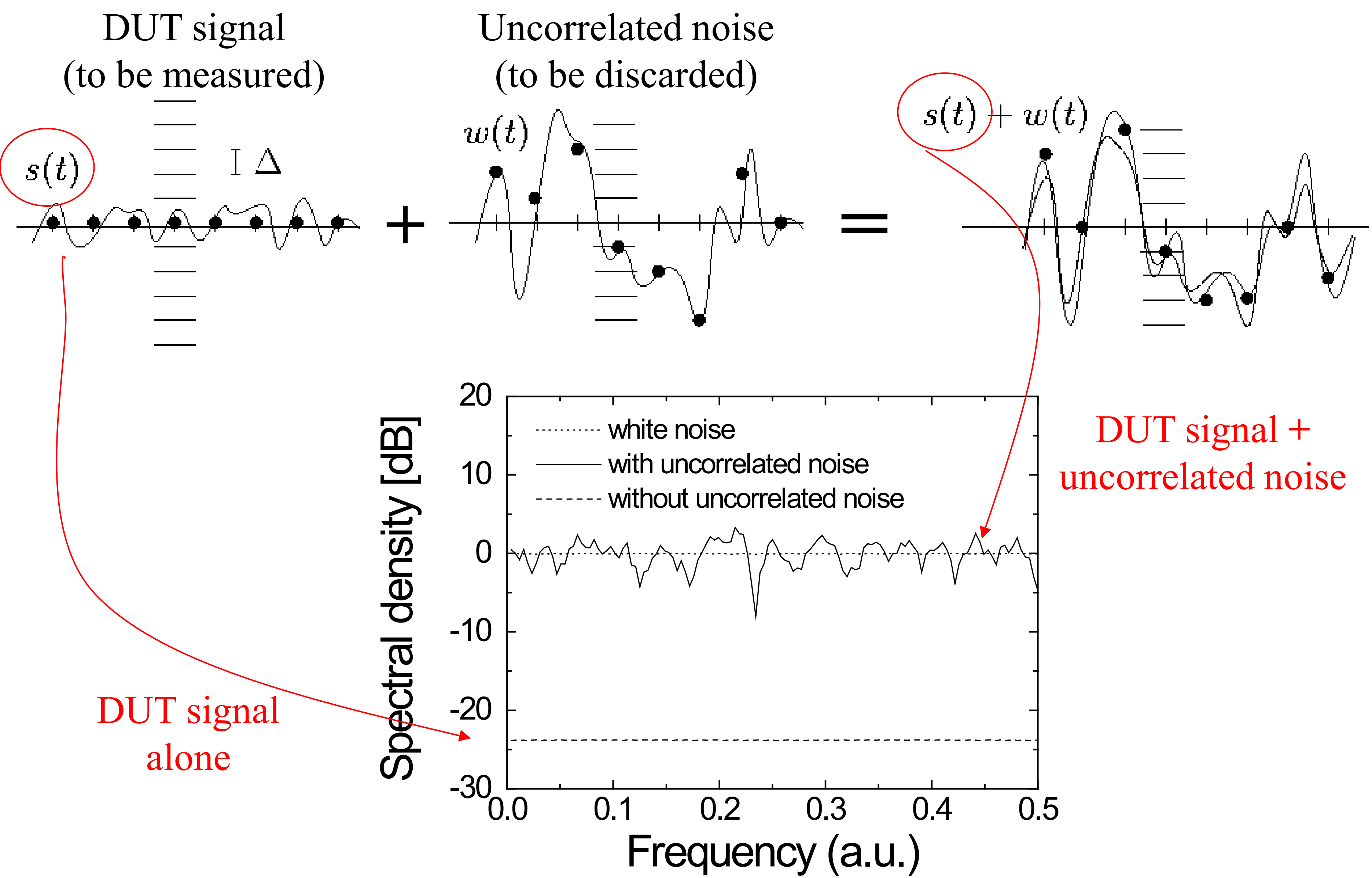
- Current noises of TIA: **uncorrelated** (is reduced by averaging)
- Voltage noise of TIA: **partly correlated** (is not reduced and sets the sensitivity limit) **partly uncorrelated** (is reduced by averaging)

Residual correlations : voltage scheme



- Current noise : **correlated**
- Voltage noise : **uncorrelated**

Suppression of the quantisation noise



Bandwidth limitations

~ mHz

- Very long measurement time (stationary noise over a long period)
- High stability of the DUT, of the amplifiers and of the connections
- Stability in the set-up temperature

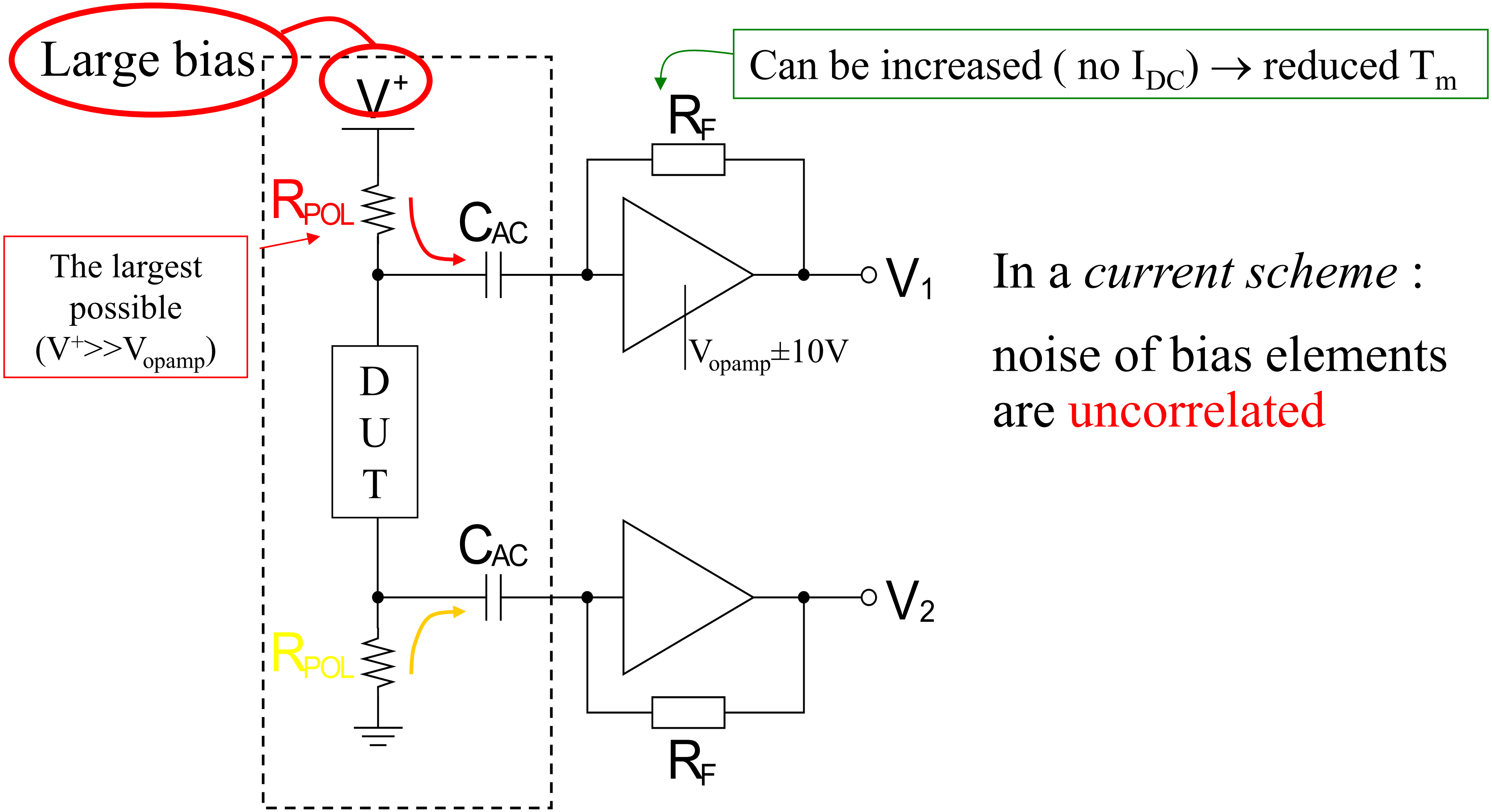
~ MHz

- Set by residual correlation through the DUT (C_D=10pF) and by parasitics (C_i=10pF)
- and by the low noise amplifier bandwidth.
- Needs high sampling frequency in digital mode

~ GHz

- Reduction of C_D and parasitics (integrated DUT and amplifiers)
- High bandwidth amplifiers (voltage scheme favored).
- Heterodyne system

AC coupling of the DUT – current scheme

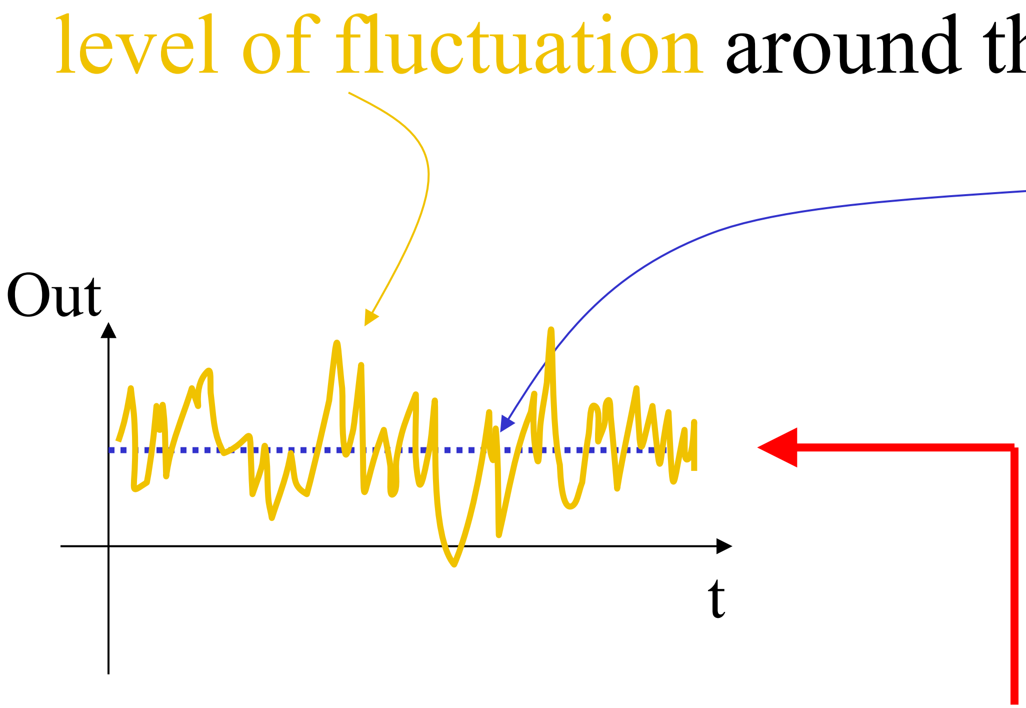


In a *current scheme* :

noise of bias elements are uncorrelated

Example: V^+=100V instead of V_{opamp}=10 → $S_R(f)/5$ → $T_m/25$!

Accuracy of the instrument



Instrument *accuracy* is limited by the precision in the calculation of :

- the system gain
- the system frequency response